SM223 – Calculus III with Optimization
Assoc. Prof. Nelson Uhan

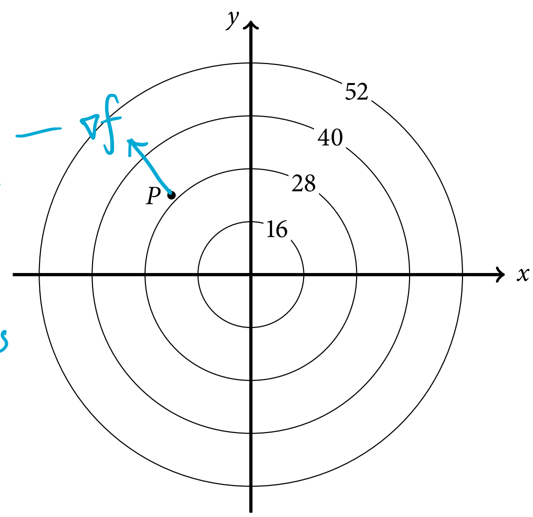
Fall 2017

# Review Quiz 3

**Instructions.** You have 20 minutes to complete this review quiz. You <u>may</u> use your calculator. You may not use any other materials. Put your answers on the separate answer form provided.

1. Consider the following contour plot of the surface $z = f(x, y)$:

$\nabla f$ is perpendicular — $\nabla f$
to the level curves
of $f$, pointing in
the direction of
<u>increasing</u> $f$ values




The gradient for this surface at point $P$ is in the direction of:

(a) $-\vec{i} - \vec{j}$
(b) $\vec{i} + \vec{j}$
(c) $\vec{i} - \vec{j}$
(d) $\vec{j}$
(e) $-\vec{i} + \vec{j}$

2. Consider the contour plot of the surface $z = f(x, y)$ in Problem 1. Which statement is true regarding $f_x$ and $f_y$ at point $P$?

(a) $f_x < 0$ and $f_y < 0$
(b) $f_x > 0$ and $f_y < 0$
(c) $f_x < 0$ and $f_y > 0$
(d) $f_x < 0$ and $f_y > 0$
(e) cannot determine – not enough information

3. An equation for the plane tangent to the surface $z = e^{2x+y}$ at the point $(0, 0, 1)$ is

(a) $z = 1$
(b) $z = x + y + 1$
(c) $z = x + 2y + 1$
(d) $z = 2x + y + 1$
(e) $z = 2xe^{2x+y} + ye^{2x+y} + 1$

surface $F(x,y,z) = k$
tangent plane at $(x_0,y_0,z_0)$:
$$F_x(x_0,y_0,z_0)(x-x_0) + F_y(x_0,y_0,z_0)(y-y_0) + F_z(x_0,y_0,z_0)(z-z_0) = 0$$

Here, $F(x,y,z) = e^{2x+y} - z \Rightarrow \nabla F(x,y,z) = \langle 2e^{2x+y}, e^{2x+y}, -1 \rangle$
$$\nabla F(0,0,1) = \langle 2, 1, -1 \rangle$$

$\Rightarrow$ tangent plane: $2x + y - (z-1) = 0$

4. Let $f(x, y, z) = x^2y + yz$. The directional derivative of $f$ at the point $(3, -2, 4)$ in the direction of the unit vector $\langle 2/3, 2/3, -1/3\rangle$ is:

   (a) $-26$
   (b) $-2/3$
   (c) $0$
   (d) $4/3$
   (e) $46/9$

$\nabla f(x,y,z) = \langle 2xy,\ x^2 + z,\ y \rangle$

$\nabla f(3,-2,4) = \langle -12, 13, -2 \rangle$

$\Rightarrow D_{\vec{u}} f(3,-2,4) = \langle -12, 13, -2 \rangle \cdot \langle \frac{2}{3}, \frac{2}{3}, -\frac{1}{3} \rangle = \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$

5. If $f_x(1, 2) = f_y(1, 2) = 0$, $f_{xx}(1, 2) = 3$, $f_{yy}(1, 2) = 5$, and $f_{xy}(1, 2) = 2$, then:

   (a) $f$ has a local minimum at $(1, 2)$
   (b) $f$ has a local maximum at $(1, 2)$
   (c) $f$ has a saddle point at $(1, 2)$
   (d) $f$ has neither a local extreme point nor a saddle point at $(1, 2)$
   (e) There is not enough information to determine the behavior of $f$ at $(1, 2)$

$D(x,y) = f_{xx}(x,y)\, f_{yy}(x,y) - f_{xy}(x,y)^2$

$$\left.\begin{array}{l} \Rightarrow D(1,2) = 15 - 4 = 11 > 0 \\ f_{xx}(1,2) = 3 > 0 \end{array}\right\} \text{local minimum at } (1,2)$$